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III Semester B.Sc. Degree Examination, March/April - 2021

MATHEMATICS

(CBCS Semester)

Paper : III

Time : 3 Hours

Maximum Marks : 70

**Instructions to Candidates:**

Answer all questions.

**PART - A**

Answer any five questions.

(5×2=10)

1. a) Write the orders of the elements of the group  $\{1, w, w^2\}$  with respect to multiplication.
- b) Define right coset and left coset of a group.
- c) Discuss the convergence of the sequence  $\frac{\log n}{n}$ .
- d) Test the convergence of the series  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ .
- e) State Roabe's test for convergence of series.
- f) Prove that every differentiable function is continuous.
- g) Find the value of 'c', using Rolle's theorem for the function  $f(x) = x^2 - 6x + 8$  in  $[2,4]$ .
- h) Evaluate  $\lim_{x \rightarrow \infty} \frac{x - \sin x}{x^3}$

**PART - B**

Answer any One full question.

(1×15=15)

2. a) If 'a' is an element of order 'n' of a group G, 'e' is the identify, then for some positive integer m,  $a^m = e$  if and only if 'n' is the divisor of m.
- b) Find all the generators of the cyclic group of order 8.
- c) State and prove Lagrange's theorem for finite groups.

(OR)

[P.T.O.]



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3. a) Find the order of each element of the group  $G = \{0, 1, 2, 3, 4, 5\}$  under  $\oplus_6$ .
- b) Prove that every subgroup of a cyclic group is cyclic.
- c) Find all the distinct right cosets of the subgroup  $H = \{0, 4, 8\}$  in  $(Z_{12}, \oplus_{12})$ .

**PART - C**

Answer two full questions.

(2×15=30)

4. a) If  $\lim_{n \rightarrow \infty} a_n = a$ ,  $\lim_{n \rightarrow \infty} b_n = b$ . Prove that  $\lim_{n \rightarrow \infty} a_n b_n = ab$ .
- b) Discuss the convergence of the sequences

i.  $\left\{ \frac{(-1)^{n-1}}{n} \right\}$

ii.  $\left\{ \left( \frac{n+1}{n} \right) \frac{3n^2}{n+1} \right\}$

- c) Find the limit of the sequence 0.4, 0.44, 0.444, .....

(OR)

5. a) Prove that every convergent sequence is bounded.
- b) Prove that the sequence  $\left\{ \frac{3n+4}{2n+1} \right\}$  is monotonically decreasing and converges to  $\frac{3}{2}$ .
- c) Prove that a monotonically increasing sequence which is bounded above is convergent.
6. a) State and prove Cauchy's root test for the convergence of series of positive terms.

- b) Discuss the convergence of the series  $\frac{1}{3} + \frac{2^3}{3^2} + \frac{3^3}{3^3} + \frac{4^3}{3^4} + \dots$

- c) Find the sum of infinity of the series

$$1 + \frac{4}{6} + \frac{4.5}{6.9} + \frac{4.5.6}{6.9.12} + \dots$$

(OR)



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7. a) Discuss the convergence of the series

$$\frac{1}{\sqrt{1+\sqrt{2}}} + \frac{1}{\sqrt{2+\sqrt{3}}} + \frac{1}{\sqrt{3+\sqrt{4}}} + \dots$$

- b) Test the absolute, conditional convergence of the series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$

- c) Find the sum to infinity of the series

$$1 + \frac{1+5}{2!} + \frac{1+5+5^2}{3!} + \dots$$

**PART - D**

Answer one full question.

(1×15=15)

8. a) Discuss the differentiability of the function at  $x = 0$  if  $f(x) = \begin{cases} 1+2x & \text{for } x \leq 0 \\ 1-3x & \text{for } x > 0 \end{cases}$
- b) State and prove Lagrange's Mean value Theorem.
- c) Evaluate

i.  $\lim_{x \rightarrow 0} \frac{\log(\tan x)}{\log(x)}$

ii.  $\lim_{x \rightarrow \frac{\pi}{2}} (\sec x - \tan x)$

(OR)

9. a) Discuss the continuity of the function  $f(x)$  at  $x = 2$  if  $f(x) = \begin{cases} 1+x & \text{for } x \leq 2 \\ 5-x & \text{for } x > 2 \end{cases}$
- b) Find the Maclaurin's series expansion of  $\log(\sec x)$  upto the term containing  $x^4$ .
- c) Evaluate

i.  $\lim_{x \rightarrow 0} \frac{a^x - b^x}{x}$

ii.  $\lim_{x \rightarrow 0} (1 + \sin x)^{\cot x}$

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